Bilkent University
Department of Mathematics

Senior Projects

Math 491/492

2024-Fall

Course Committee: Ö. Ünlü and G. Yıldırım

Course Coordinator: G. Yıldırım

Project Presentations

Wednesday, December 25, in the seminar room (SA-141)

10:00–10:30: Adem Eren Uyanık - (Berktav)
- 2-Dimensional Topological Quantum Field Theories

10:30–11:00: Behzat Deniz Özyörük -(Okay)
- Simplicial Distributions as a Convex Monoid

Break 11:00-11:15

11:15–11:45: Alper Akyüz- (Özsarı)
- Analysis of the Heat Equation with Wentzel Boundary Conditions Using Fokas Transform

11:45–12:15: İdil Bahar Ongun - (Yıldırım)
- Stochastic Trace Estimation

Seminar Talks

Number Theory Seminar-December 24: Muhammed Gökmen - (Parry)
- The Average Order of the Prime Omega Function

Analysis Seminar-December 17: Ziya Kağan Akman - (Saldı) (Math 492)
- Discrete-Time Markov Decision Processes

2-Dimensional Topological Quantum Field Theories

Adem Eren Uyanık

Supervisor: Kadri İlker Berktav

The flow of inspiration has been mainly from mathematics to physics throughout the last century. In this talk, I will present one of the few exceptions: the *Topological Quantum Field Theories* (TQFTs). In this regard, I will first give the necessary category theoretical background and then provide the formal definition of TQFTs. Later, I will focus on 2-TQFTs as a particular case and briefly discuss their classification by means of *Frobenious algebras*.

Simplicial Distributions as a Convex Monoid

Behzat Deniz Özyörük

Supervisor: Cihan Okay

Probability distributions obtained from experiments in quantum mechanics do not always admit a joint probability distribution. These distributions are called contextual distributions. Simplicial distributions provide a topological framework for analyzing contextuality based on simplicial sets. In this project, a monoid-theoretic characterization of contextuality is studied in the framework of simplicial distributions by utilizing tools from category theory.

Analysis of the Heat Equation with Wentzel Boundary Conditions Using Fokas Transform

Alper Akyüz

Supervisor: Türker Özsarı

In this project, we will study the well posedness of the linear heat equation with inhomogeneous Wentzell boundary conditions. We will review the unified transform solution of the Initial Boundary Value Problem in the half-line as established in [1]. Using the integral representation established, we will prove well-posedness in the Hadamard space $C([0,T];H^s(\mathbb{R}_+))$ of the Initial Boundary Value Problem (IBVP) in the half-line setting, a new result in the literature within the framework of UTM based analysis.

[1] D. Mantzavinos and A. S. Fokas. The unified method for the heat equation: I. Non-separable boundary conditions and non-local constraints in one dimension. European J. Appl. Math., 24(6):857–886, 2013.

Stochastic Trace Estimation

İdil Bahar Ongun

Supervisor: Gökhan Yıldırım

Stochastic trace estimation is a computational method for approximating the trace of large-scale matrices, a critical quantity in fields such as machine learning, network science, and quantum mechanics. This project focuses on the Girard-Hutchinson estimator, which efficiently approximates traces using randomized matrix-vector products, bypassing the need for explicit matrix computations. The project examines the performance of three types of random test vectors—Gaussian, Rademacher, and spherical—evaluating their impact on the estimator's variance and computational efficiency.

A significant application of this method is triangle counting in large graphs, an important task in network analysis. Triangles in networks provide insights into clustering and connectivity, key metrics in areas like social networks and biology. By approximating the trace of the cube of the adjacency matrix, the number of triangles can be estimated efficiently even for massive graphs where direct computation is impractical. To ensure reliability, we study probabilistic concentration bounds like Bernstein's and Hoeffding's inequalities, establishing rigorous guarantees for the required sample size. Numerical experiments on Erdős-Rényi random graphs examine the effectiveness of the proposed approach.

The Average Order of the Prime Omega Function

Muhammed Gökmen

Supervisor: Tomos Parry

Write

$$\pi(x) := \sum_{p \le x} 1$$

for the number of primes up to x. The Prime Number Theorem (PNT) gives us an approximation for this number, precisely it says

$$\pi(x) = \operatorname{li}(x) + \mathcal{O}\left(xe^{-\sqrt{\log x}}\right) \qquad \operatorname{li}(x) := \int_2^x \frac{dt}{\ln(t)}.$$

The proof method was laid out in Riemann's famous memoir and uses the complex analytic properties of the Riemann zeta function $\zeta(s)$. For example, the main term in the above formula comes from the Residue Theorem, picking up a simple residue of $\zeta(s)$ at $s = 1$.

Write $\omega(n)$ for the number of (distinct) prime factors of a number and write

$$W(x) := \sum_{n \le x} \omega(n).$$

In this talk we discuss the corresponding PNT statement for $W(x)$, leading us to look at $\log \zeta(s)$, whose logarithmic singularity at $s = 1$ means the complex analysis study is more awkward than that of $\zeta(s)$.

Discrete-Time Markov Decision Processes (math 492)

Ziya Kağan Akman

Supervisor: Naci Saldı

The aim of this project is to comprehend the essential theory of discrete-time Markov Decision Processes (MDP). We commence with defining MDP and examining its stochastic properties. Then, we proceed with describing policies and classifying them. Later, we will make the connection between policies and MDPs via canonical construction. Consequently, we explore how to minimize the performance criterion and find an optimal policy in the settings of both finite-horizon problems and discounted infinite-horizon problems. Finally, we end our discussion by considering some examples of finite-horizon and discounted infinite-horizon problems.